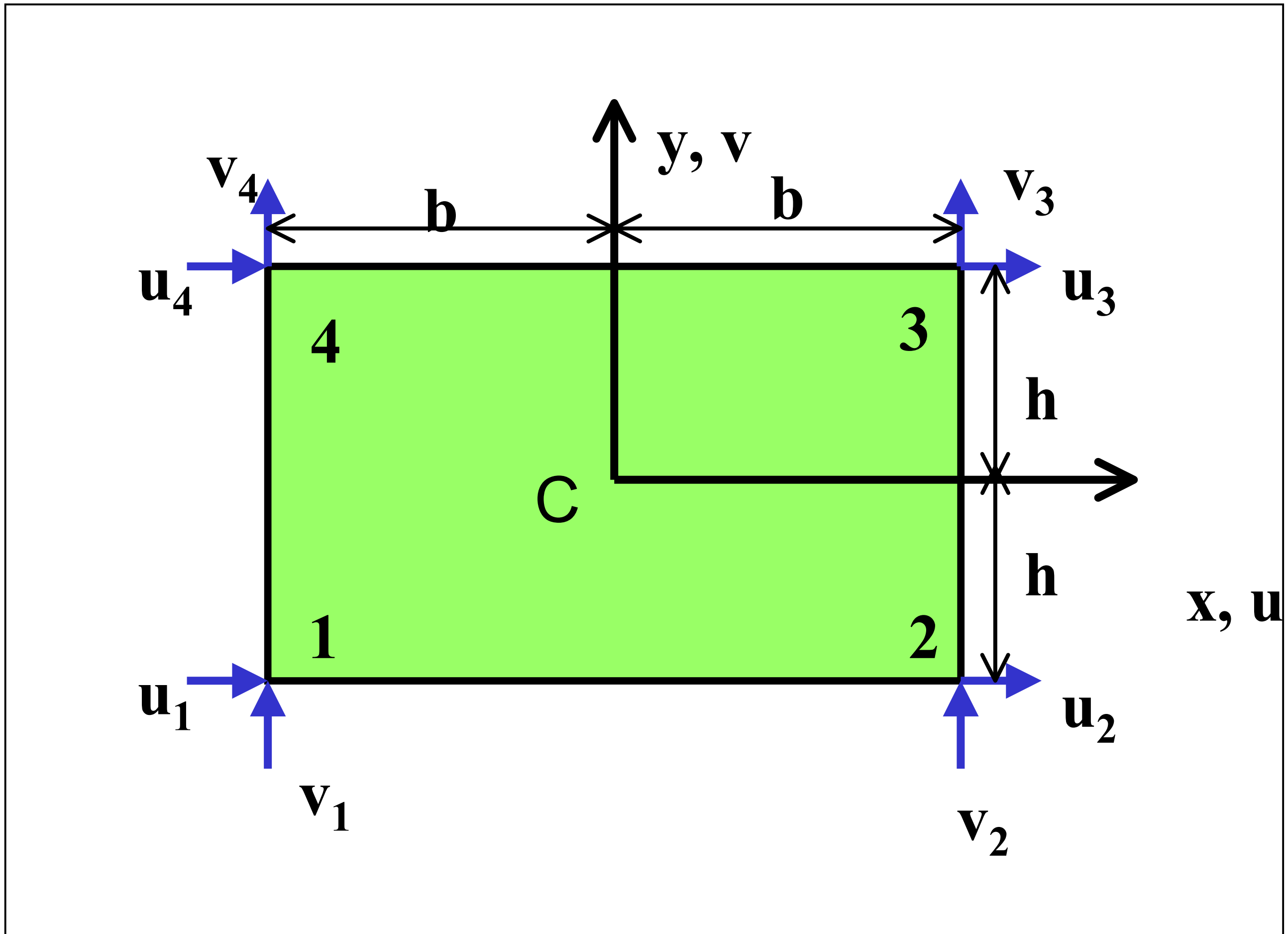
Formularea izoparametrică

Termenul de izoparametric se referă la faptul ca aceeaşi funcţie care descrie forma elementului este utilizată şi pentru definirea deplasărilor. Sistemul de coordonate utilizat este cel natural - intrinsec, parametric. Exemplificarea se face pentru un element tip arie cu 4 noduri -(quad).

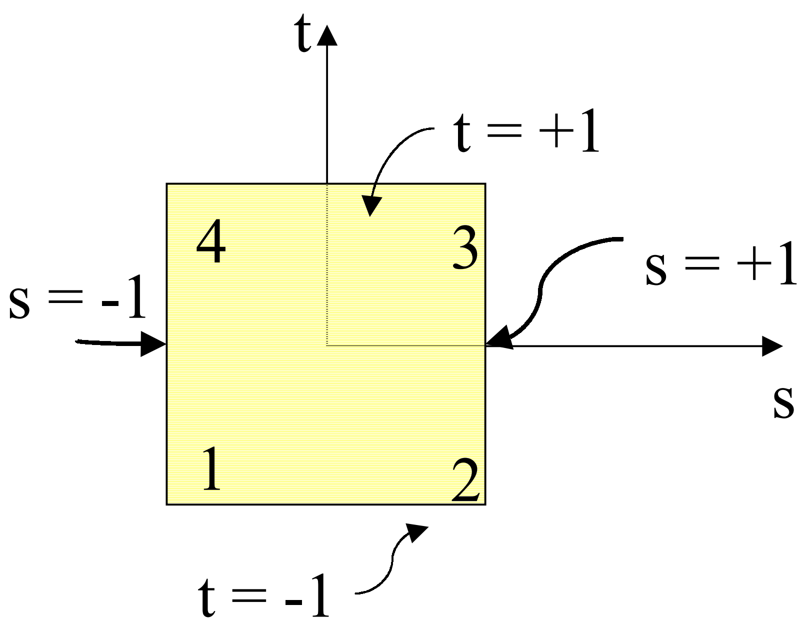
Pasul 1 - abstractizarea

Se consideră un element rectangular, în ipoteza stării plane de deformaţii.



$$\{\delta\} = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

Formularea parametrică



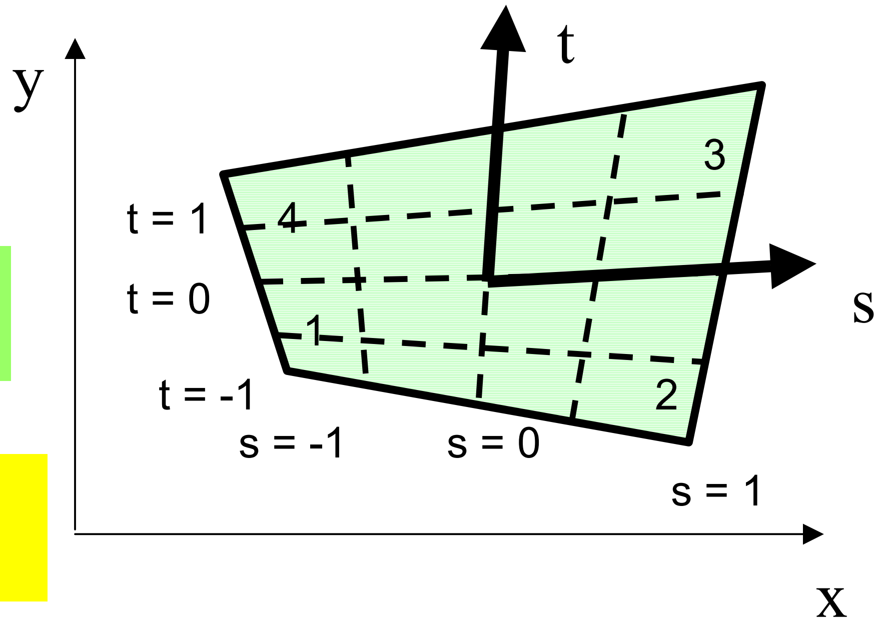
Pentru dreptunghi:

$$x = x_c + b\,s$$

$$y = y_c + h\,t$$

$$Nodurile\ sunt\ (\pm 1, \pm 1)\ in\ spatiul\ \mathrm{s - t}$$

Elementul în coordonate naturale



Formularea parametrică se poate extinde şi la configuraţii neregulate

Nu mai este vorba de o simplă rotaţie a sistemului de coordonate!

Relaţia ce defineşte legătura între cooronatele în sistemul x,y şi cele naturale , în spaţiul s,t se scrie:

$$\mathbf{x = a_1 + a_2 s + a_3 t + a_4 st}$$

$$\mathbf{y = a_5 + a_6 s + a_7 t + a_8 st}$$

$$\mathbf{x = \frac{1}{4}\left(\begin{array}{l}(1-s)(1-t)x_1 + (1+s)(1-t)x_2 \\ \qquad + (1+s)(1+t)x_3 + (1-s)(1+t)x_4\end{array}\right)}$$

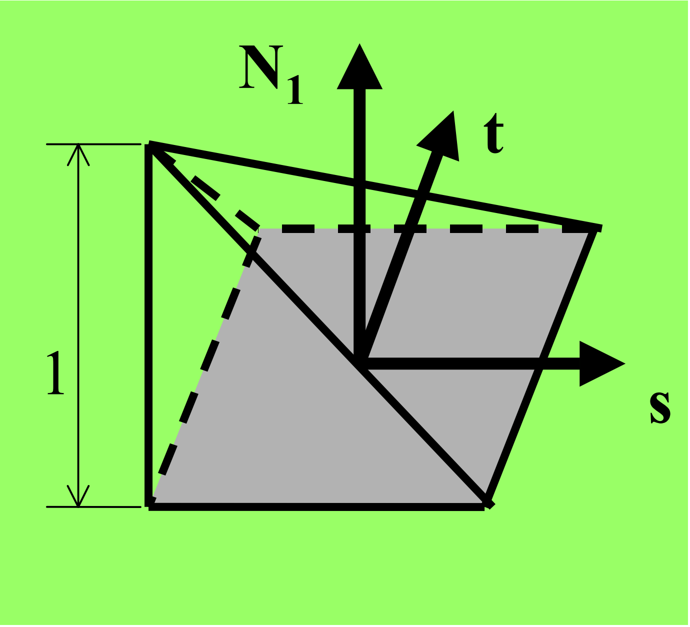
$$\mathbf{y = \frac{1}{4}\left(\begin{array}{l}(1-s)(1-t)y_1 + (1+s)(1-t)y_2 \\ \qquad + (1+s)(1+t)y_3 + (1-s)(1+t)y_4\end{array}\right)}$$

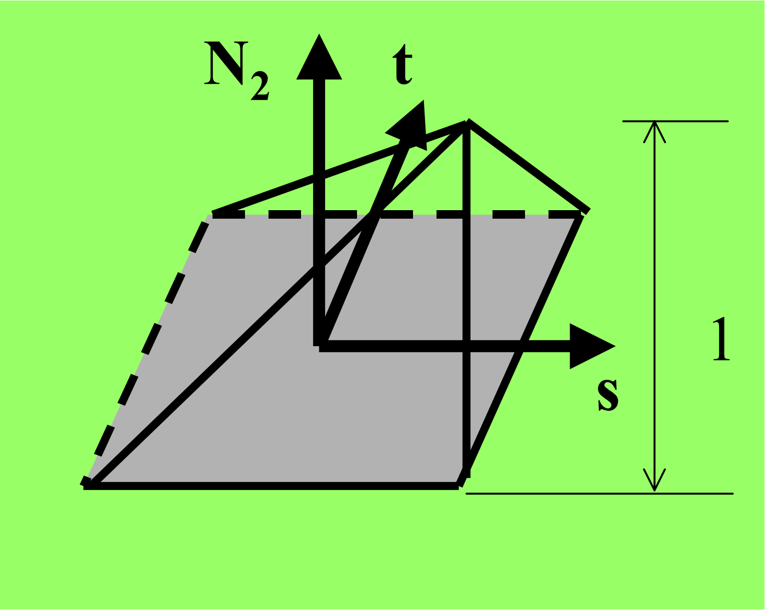
$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{y} \end{Bmatrix} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{0} & \mathbf{N}_2 & \mathbf{0} & \mathbf{N}_3 & \mathbf{0} & \mathbf{N}_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{N}_1 & \mathbf{0} & \mathbf{N}_2 & \mathbf{0} & \mathbf{N}_3 & \mathbf{0} & \mathbf{N}_4 \end{bmatrix} \begin{Bmatrix} \mathbf{x}_1 \\ \mathbf{y}_1 \\ \mathbf{x}_2 \\ \mathbf{y}_2 \\ \mathbf{x}_3 \\ \mathbf{y}_3 \\ \mathbf{x}_4 \\ \mathbf{y}_4 \end{Bmatrix}$$

cu:

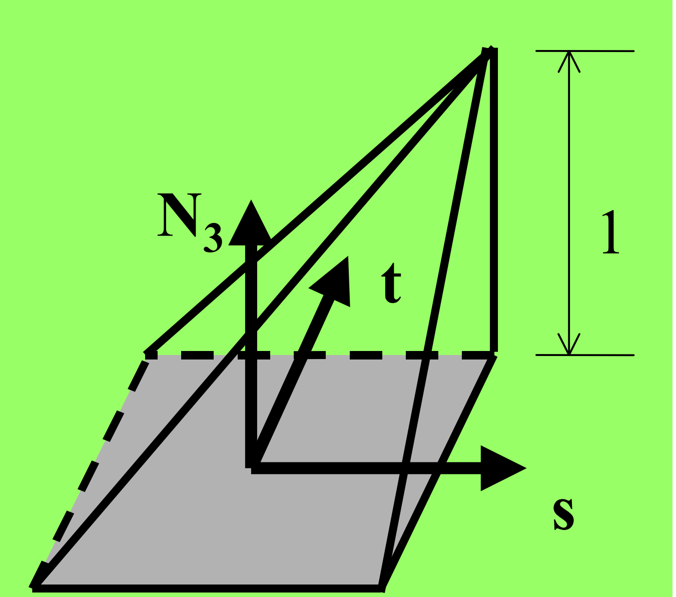
$$N_1 = \frac{(1-s)(1-t)}{4} \qquad N_2 = \frac{(1+s)(1-t)}{4}$$

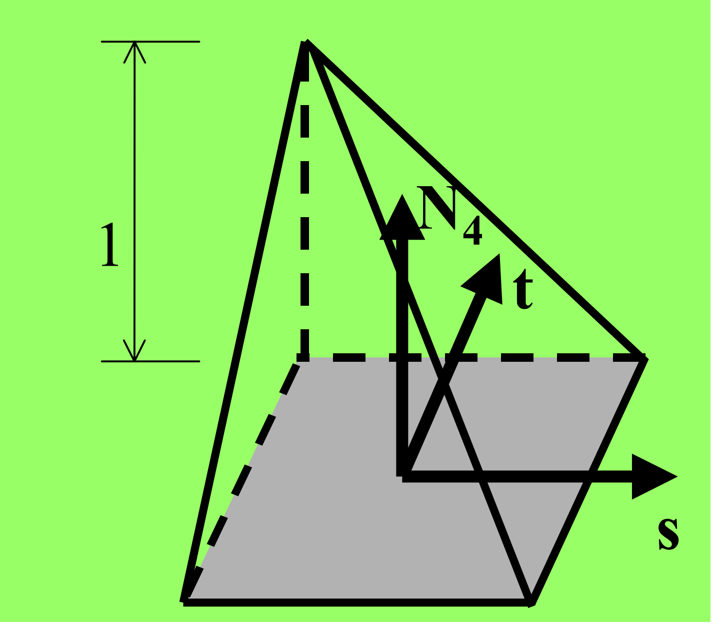
$$N_3 = \frac{(1+s)(1+t)}{4} \qquad N_4 = \frac{(1-s)(1+t)}{4}$$





$$N_1 + N_2 + N_3 + N_4 = 1$$





Pasul 2 - se alege funcţia pentru deplasări

Se folosesc aceleaşi funcţii de formă !

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

Pasul 3 - se definesc relaţiile ce leagă deformaţiile specifice de deplasări

Se caută relaţia :

$$\{\varepsilon\}=[B]\{\delta\}$$

Se porneşte de la:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u}{\partial x}\\ \dfrac{\partial v}{\partial y}\\ \dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\end{Bmatrix}$$

Care se scrie ca:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_{\mathbf{x}}\\ \varepsilon_{\mathbf{y}}\\ \gamma_{\mathbf{xy}}\end{Bmatrix}=\begin{bmatrix}\dfrac{\partial(\;)}{\partial \mathbf{x}} & \mathbf{0}\\ \mathbf{0} & \dfrac{\partial(\;)}{\partial \mathbf{y}}\\ \dfrac{\partial(\;)}{\partial \mathbf{y}} & \dfrac{\partial(\;)}{\partial \mathbf{x}}\end{bmatrix}\begin{Bmatrix}\mathbf{u}\\ \mathbf{v}\end{Bmatrix}$$

dar:

$$\frac{\partial(\ )}{\partial x}=\frac{1}{|J|}\left[\frac{\partial y}{\partial t}\frac{\partial(\ )}{\partial s}-\frac{\partial y}{\partial s}\frac{\partial(\ )}{\partial t}\right]$$

$$\frac{\partial(\ )}{\partial y}=\frac{1}{|J|}\left[\frac{\partial x}{\partial s}\frac{\partial(\ )}{\partial t}-\frac{\partial x}{\partial t}\frac{\partial(\ )}{\partial s}\right]$$

cu [J] - Jacobianul

$$[J]=\begin{bmatrix}\dfrac{\partial x}{\partial s} & \dfrac{\partial y}{\partial s}\\ \dfrac{\partial x}{\partial t} & \dfrac{\partial y}{\partial t}\end{bmatrix}$$

rezultă:

$$\{\varepsilon\} = [D'][N]\{\delta\}$$

$$[D'] = \frac{1}{|J|}\begin{bmatrix} \frac{\partial y}{\partial t}\frac{\partial(\ )}{\partial s} - \frac{\partial y}{\partial s}\frac{\partial(\ )}{\partial t} & 0 \\ 0 & \frac{\partial x}{\partial s}\frac{\partial(\ )}{\partial t} - \frac{\partial x}{\partial t}\frac{\partial(\ )}{\partial s} \\ \frac{\partial x}{\partial s}\frac{\partial(\ )}{\partial t} - \frac{\partial x}{\partial t}\frac{\partial(\ )}{\partial s} & \frac{\partial y}{\partial t}\frac{\partial(\ )}{\partial s} - \frac{\partial y}{\partial s}\frac{\partial(\ )}{\partial t} \end{bmatrix}$$

unde:

$$\underset{(3\times 8)}{[B]} = \underset{(3\times 2)}{[D']} \quad \underset{(2\times 8)}{[N]}$$

$$|J| = \frac{1}{8}\{X_c\}^T \begin{bmatrix} 0 & 1-t & t-s & s-1 \\ t-1 & 0 & s+1 & -s-t \\ s-t & -s-1 & 0 & t+1 \\ 1-s & s+t & -t-1 & 0 \end{bmatrix} \{Y_c\}$$

cu:

$$\{X_c\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} \qquad \{Y_c\} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix}$$

Pasul 4 - se calculează matricea de rigiditate

$$[K] = \iint_A [B]^T [D]\,[B]\;dx\,dy$$

ţinând cont de relaţia:

$$\iint_A f(x,y)\,dx\,dy = \iint_A f(s,t)\,|J|\,ds\,dt$$

se scrie:

$$[K] = \int_{-1}^{1}\int_{-1}^{1} [B(s,t)]^T [D]\,[B(s,t)]\;|J(s,t)|\,ds\,dt$$

Evaluarea analitică a acestei integrale este destul de dificilă şi uzual se apelează la integrarea numerică

Metoda utilizată cel mai fecvent este metoda Gauss

$$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n} W_i f(x_i)$$

W_i sunt coeficienţii de pondere pentru valoarea funcţiei în punctele gaussiene i

Ordinul n	Abscisa x_i	Coeficientul de pondere W_i
1	0.000000000000000	2.000000000000000
2	$\pm$0.577350269189626	1.000000000000000
3	$\pm$0.577350269189626	0.555555555555555
	0.000000000000000	0.888888888888888
4	$\pm$0.861136311594053	0.347854845137454
	$\pm$0.339981043584856	0.652145154862546
1	0.	2.
2	$\pm 1/\sqrt{3}$	1.
3	$\pm\sqrt{3/5}$	$5/9$
	0.	$8/9$
4	$\pm\left(\dfrac{3+2\sqrt{1.2}}{7}\right)^{\frac{1}{2}}$	$\dfrac{1}{2}-\dfrac{1}{6\sqrt{1.2}}$
	$\pm\left(\dfrac{3-2\sqrt{1.2}}{7}\right)^{\frac{1}{2}}$	$\dfrac{1}{2}-\dfrac{1}{6\sqrt{1.2}}$

Integrala bidimensională necesară pentru evaluarea lui [K] este o extindere a relaţiilor date mai sus. Valorile pentru coordonatele punctelor pe ordonată (pe direcţia y) şi pentru coeficienţii de pondere sunt aceleaşi ca cele utilizate pentru abscisă.

Pentru calculele efective:

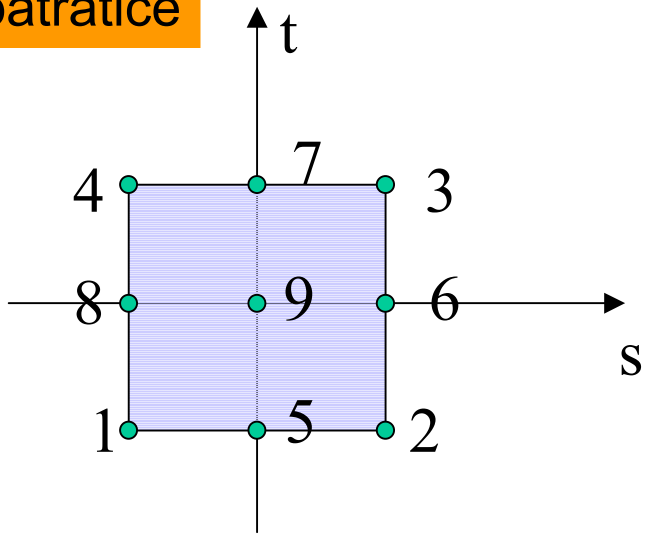
•Se evaluează fiecare termen al lui [B] şi |J| în punctele s_i ,t_i ;

•Se înmulţesc matricile;

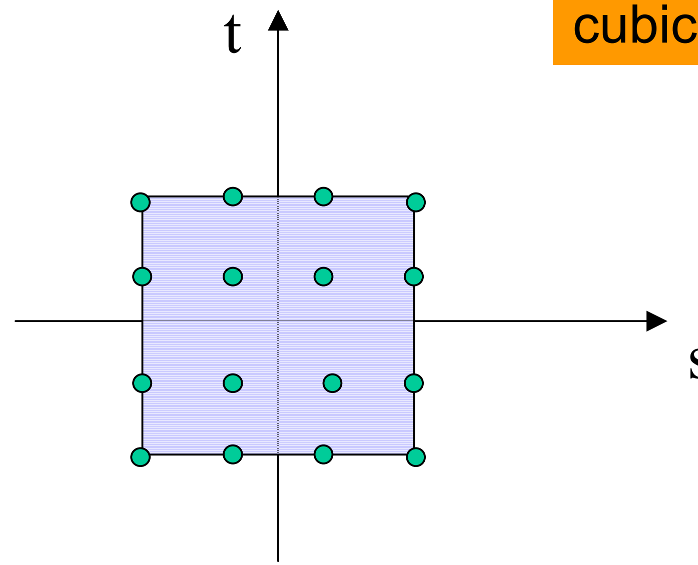
•Se multiplică cu coeficienţii de pondere;

•Se însumează apoi rezultatele.

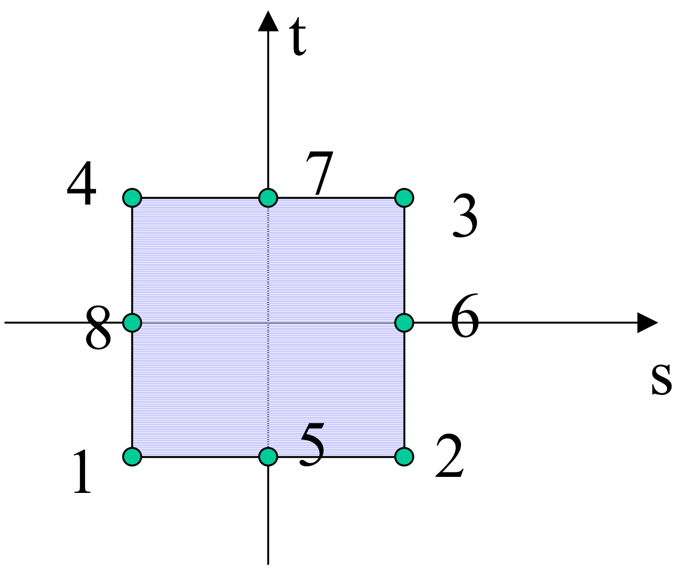
patratice

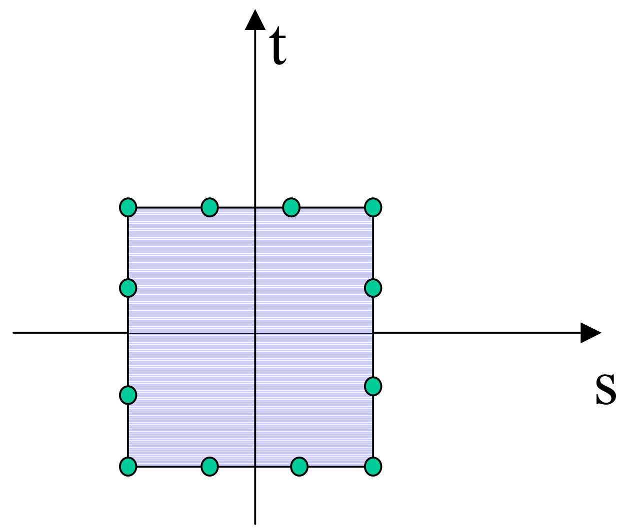


cubice



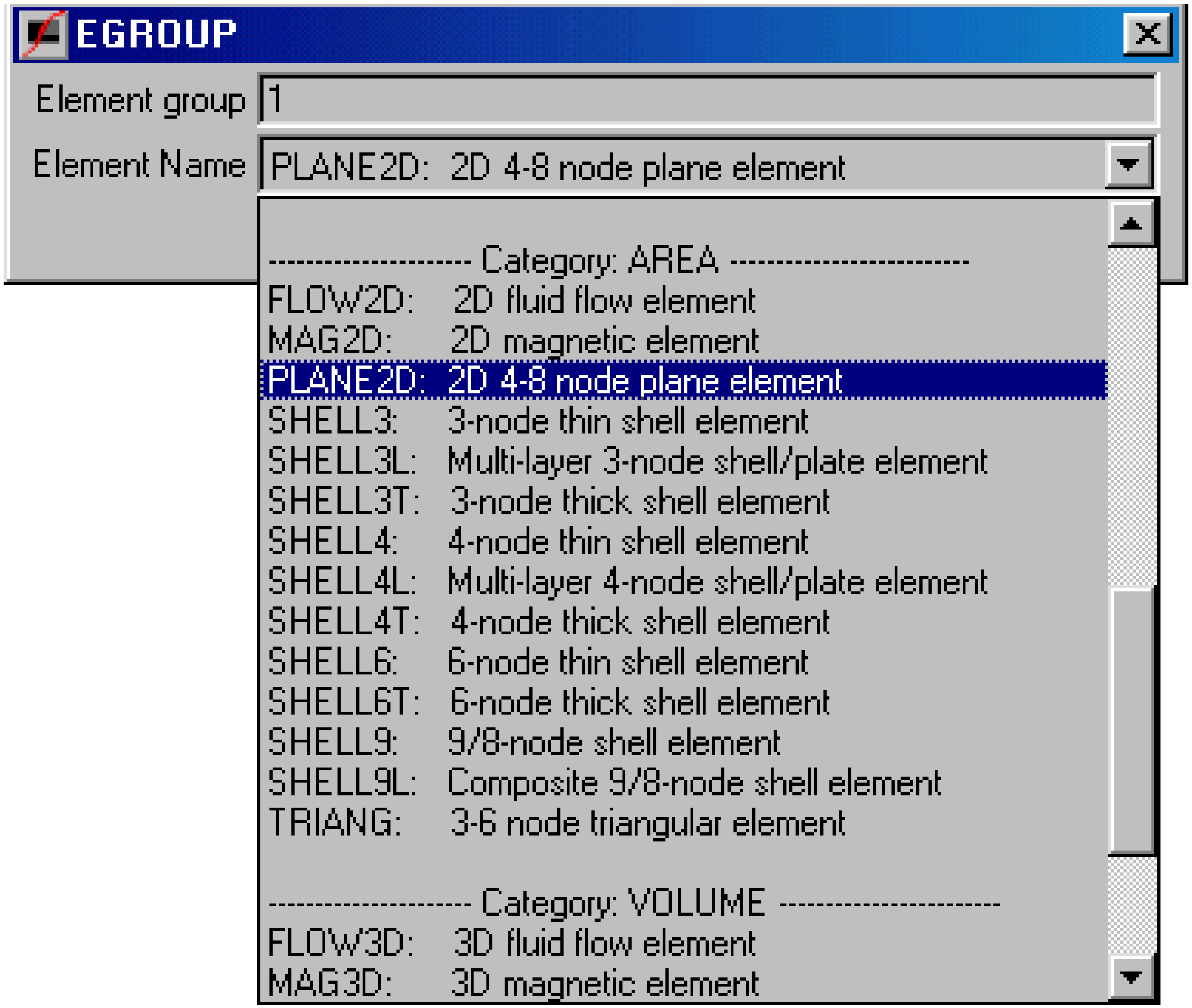
Elemente de ordin superior “Lagrange-iene”



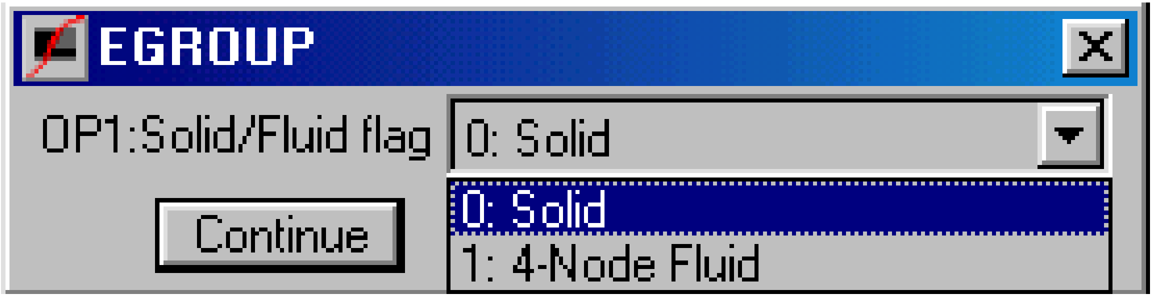


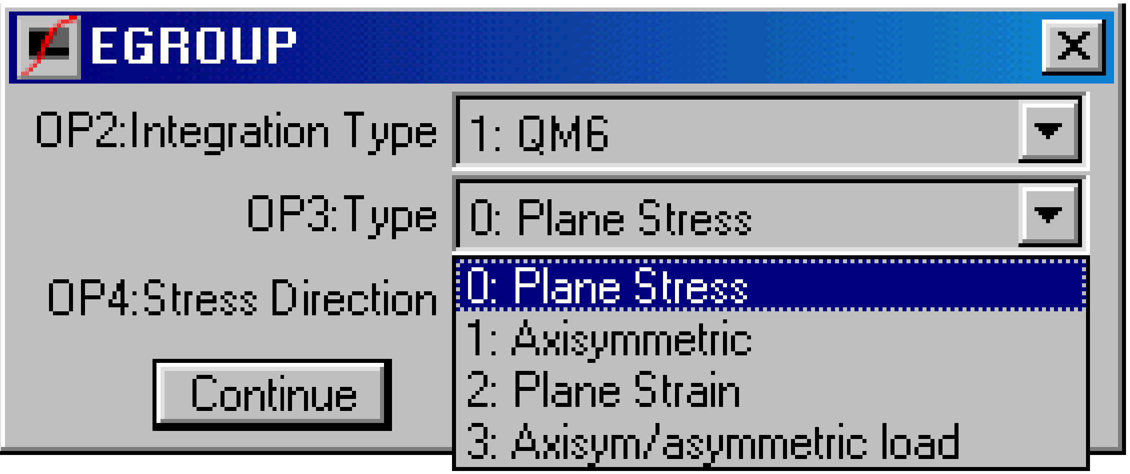
Elemente de ordin superior “Serendip-ice”

În biblioteca de elemente a programului Cosmos, elementul este denumit PLANE2D

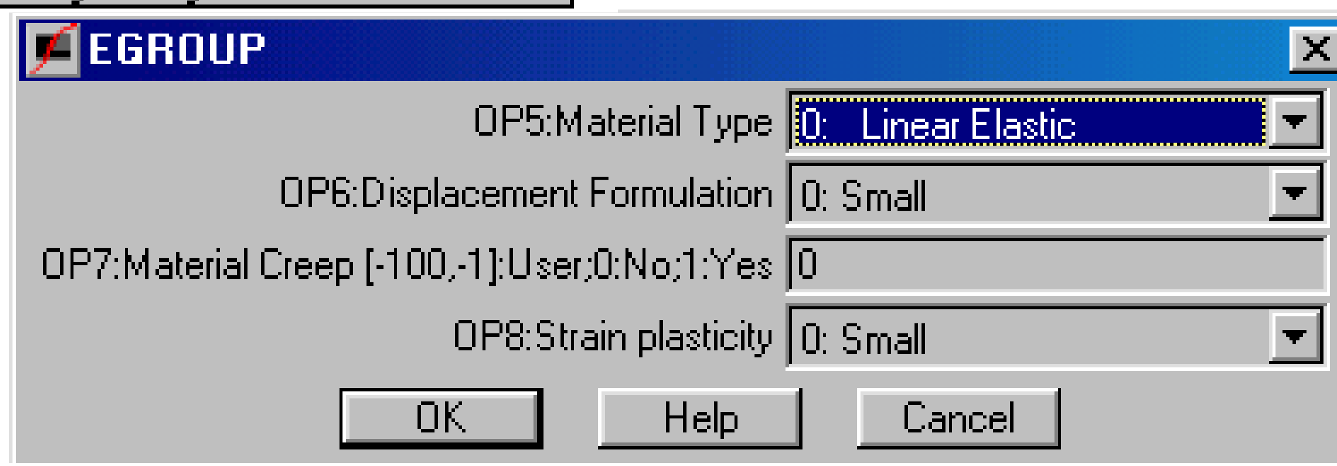


Este un element din categoria ARIE





Opţiunea 4 (OP4) permite definirea tipului de abstractizare folosit la tratarea unei probleme 3D în 2D



Proprietăţile de material ce trebuie precizate în cazul unei analize structurale sunt E şi υ

Exemplu : bimetal solicitat termic

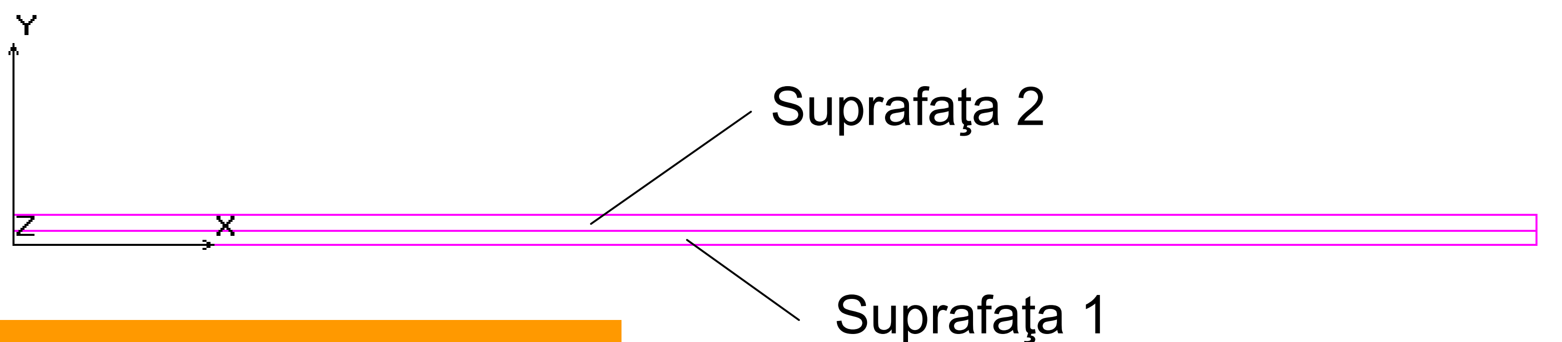
$h_1 = h_2 = 0.3$ mm; $L = 300$ mm

$E_1 = 2.1\ 10^5$ MPa; $E_2 = 2\ 10^5$ MPa

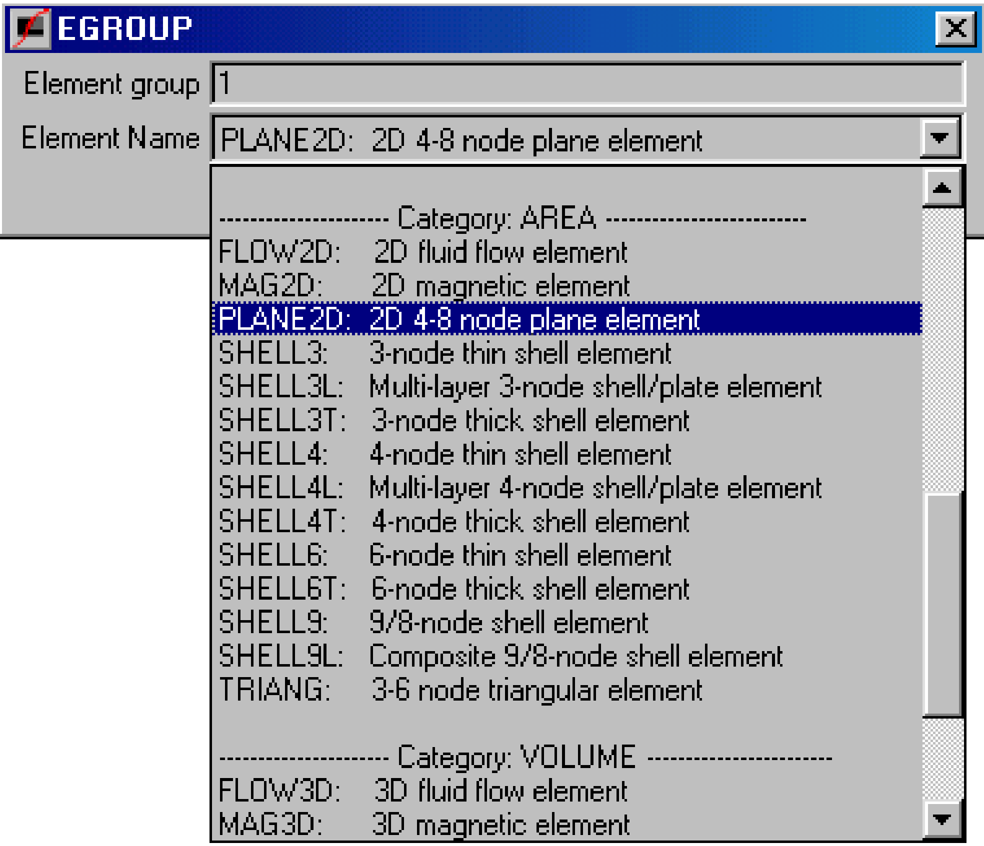
$\alpha_1 = 2\ 10^{-6}$ /°C; $\alpha_2 = 2\ 10^{-5}$ /°C

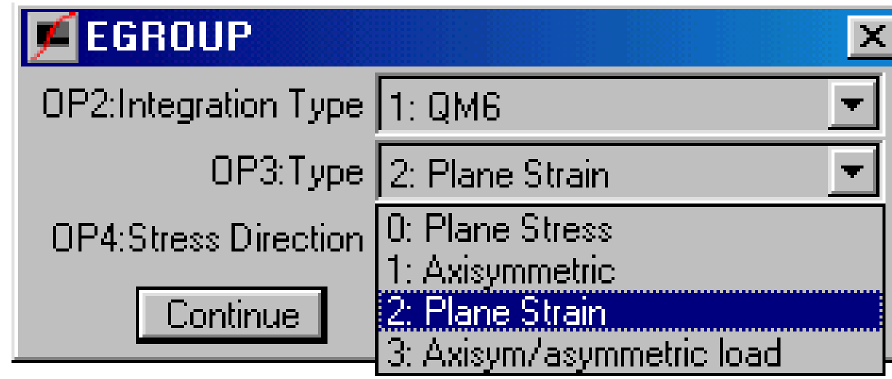
$\Delta T = 100$ °C

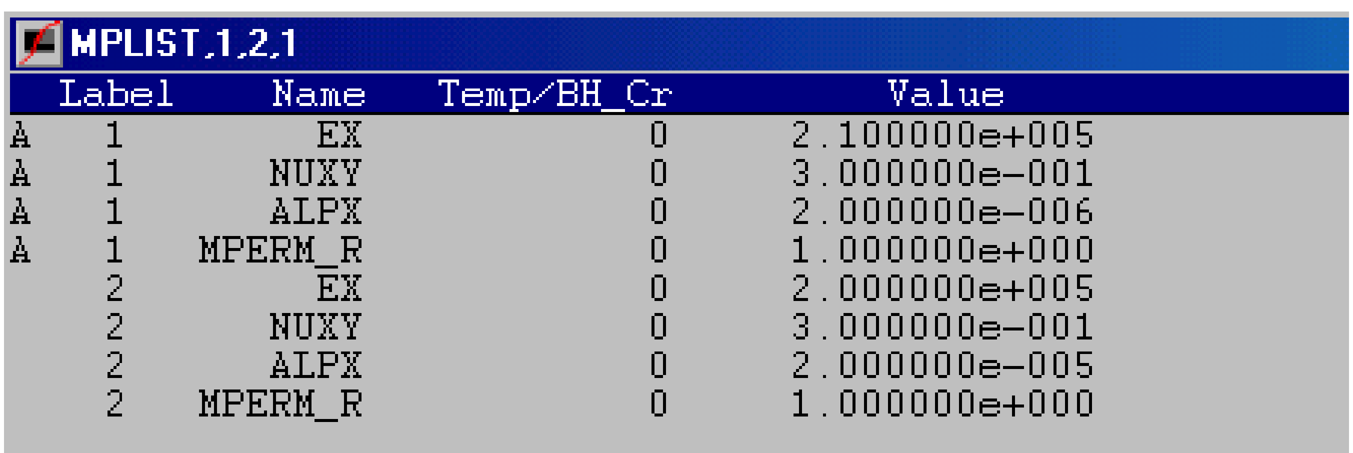
Deformaţiile specifice induse termic sunt date de relaţia: $\varepsilon = \alpha \cdot \Delta T$



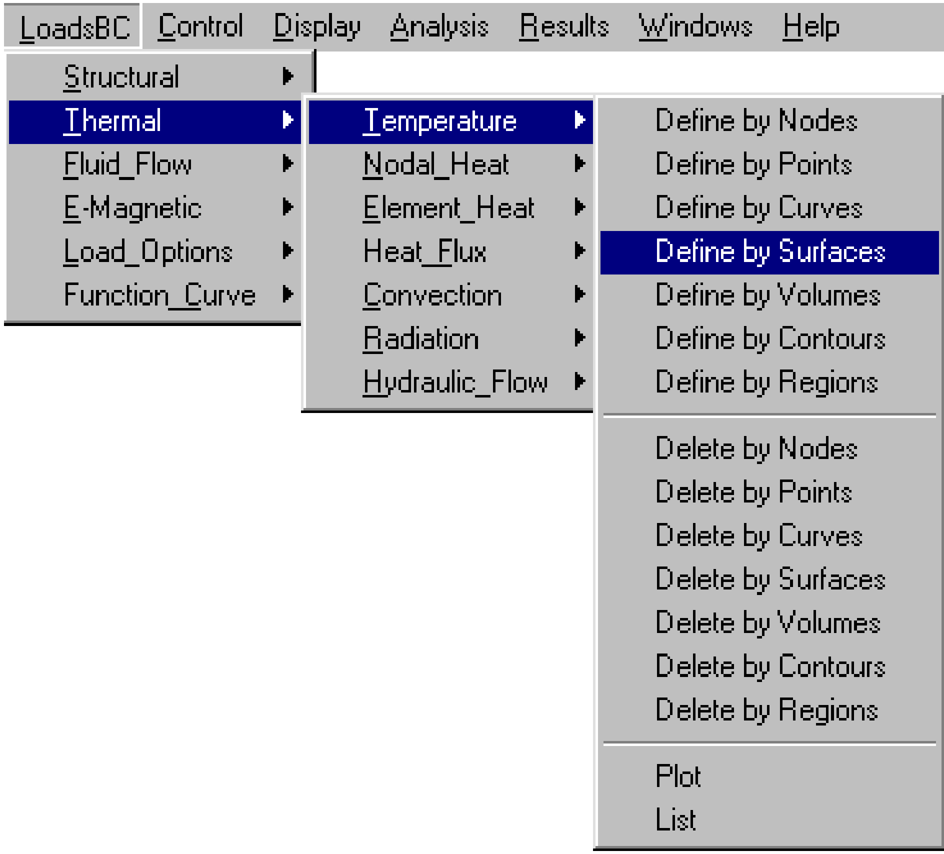
Se modelează ca o problemă de stare plană de deformaţii



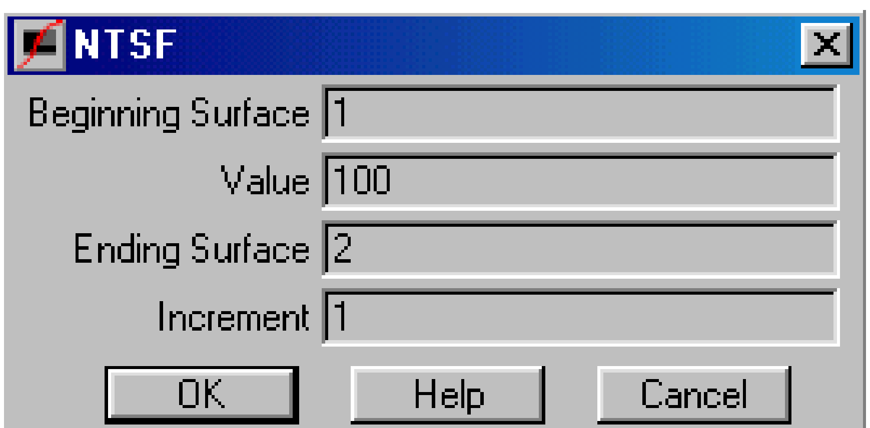


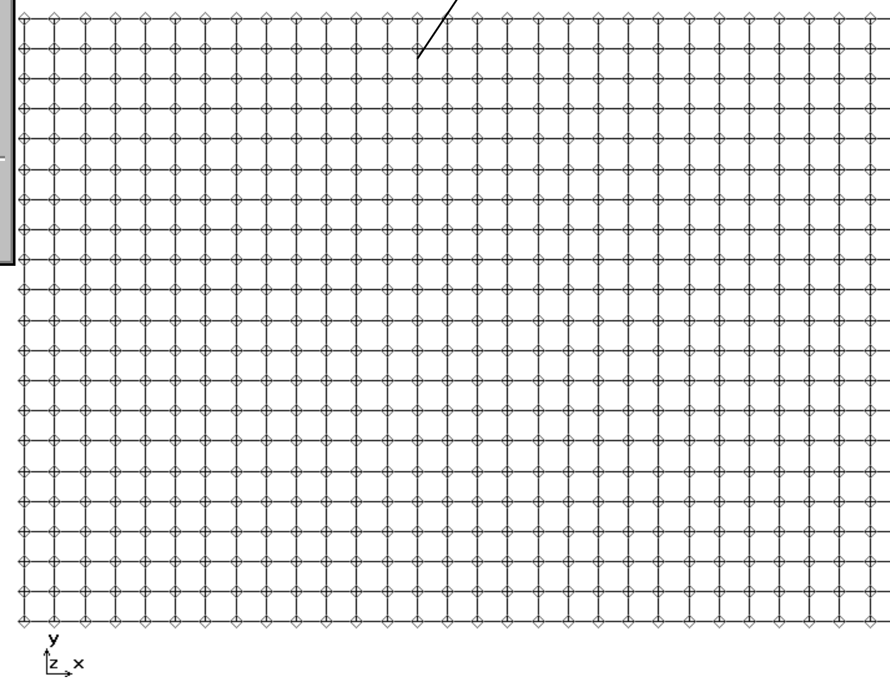

MPLIST,1,2,1

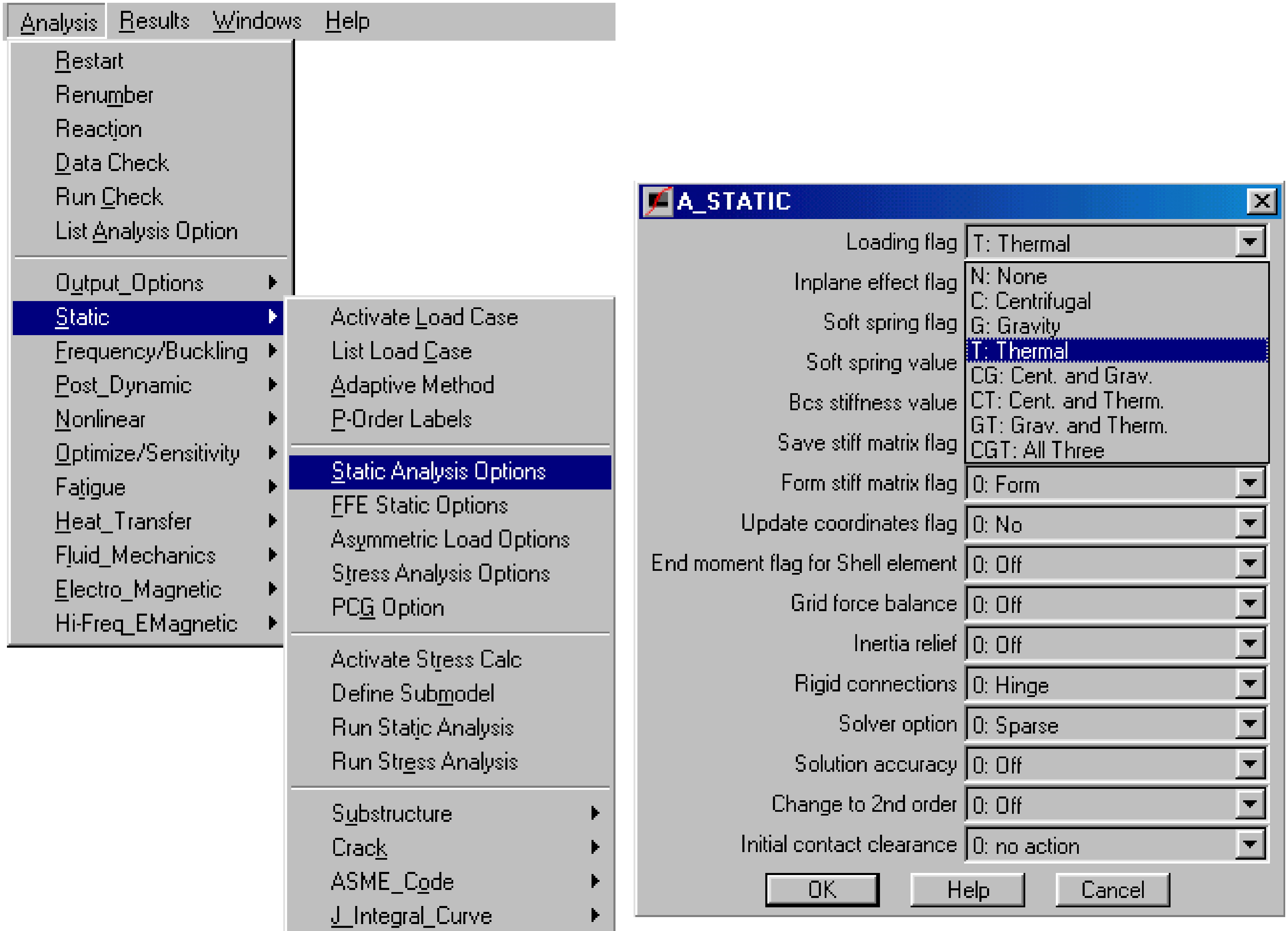
	Label	Name	Temp/BH_Cr	Value
A	1	EX	0	2.100000e+005
A	1	NUXY	0	3.000000e-001
A	1	ALPX	0	2.000000e-006
A	1	MPERM_R	0	1.000000e+000
	2	EX	0	2.000000e+005
	2	NUXY	0	3.000000e-001
	2	ALPX	0	2.000000e-005
	2	MPERM_R	0	1.000000e+000



Aplicarea încărcării termice ca o temperatură aplicată în toate nodurile aparţinând celor două suprafeţe






Analysis Results Windows Help
Restart
Renumber
Reaction
Data Check
Run Check
List Analysis Option
Output_Options
Static
Frequency/Buckling
Post_Dynamic
Nonlinear
Optimize/Sensitivity
Fatigue
Heat_Transfer
Fluid_Mechanics
Electro_Magnetic
Hi-Freq_EMagnetic
Activate Load Case
List Load Case
Adaptive Method
P-Order Labels
Static Analysis Options
FFE Static Options
Asymmetric Load Options
Stress Analysis Options
PCG Option
Activate Stress Calc
Define Submodel
Run Static Analysis
Run Stress Analysis
Substructure
Crack
ASME_Code
J_Integral_Curve
A_STATIC
Loading flag
T: Thermal
Inplane effect flag
N: None
C: Centrifugal
G: Gravity
T: Thermal
CG: Cent. and Grav.
CT: Cent. and Therm.
GT: Grav. and Therm.
CGT: All Three
Soft spring flag
Soft spring value
Bcs stiffness value
Save stiff matrix flag
Form stiff matrix flag
0: Form
Update coordinates flag
0: No
End moment flag for Shell element
0: Off
Grid force balance
0: Off
Inertia relief
0: Off
Rigid connections
0: Hinge
Solver option
0: Sparse
Solution accuracy
0: Off
Change to 2nd order
0: Off
Initial contact clearance
0: no action
OK
Help
Cancel

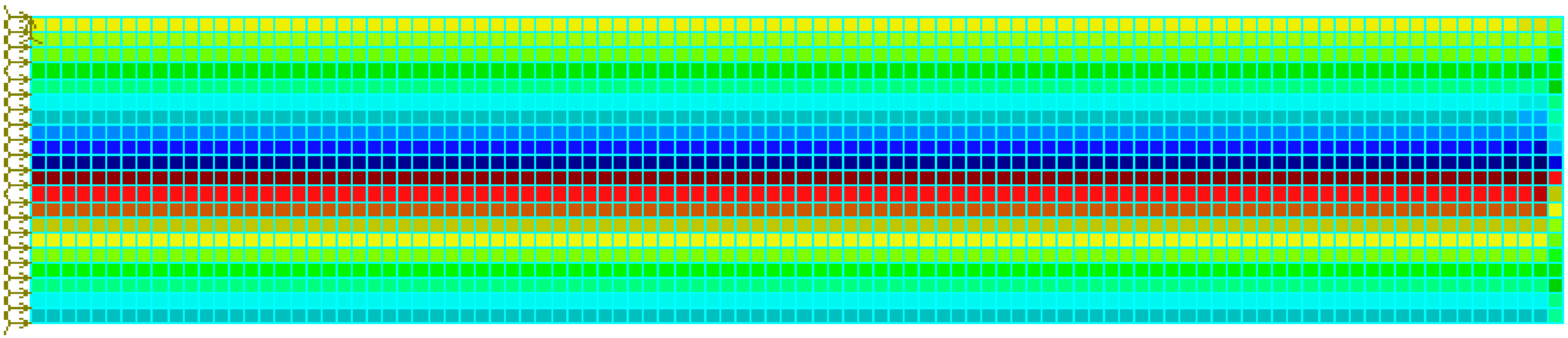
Rezultate: - modul de deformare şi deplasările maxime (la capătul liber)

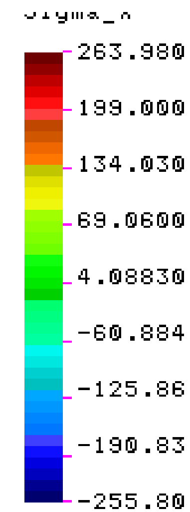
DISMAX,1,URES,1,0,1

Load case 1

Node	Disp_Res
1111	2.63379
1110	2.63368
1109	2.63358
1108	2.6335
1107	2.63343
1106	2.63337
1105	2.63332
1104	2.63329
1103	2.63326
1102	2.63325
1101	2.63325
2221	2.63316
2220	2.63309
2219	2.63303
2218	2.63298
2212	2.63294
2217	2.63294
2213	2.63292
2216	2.63292
2214	2.63291
2215	2.63291

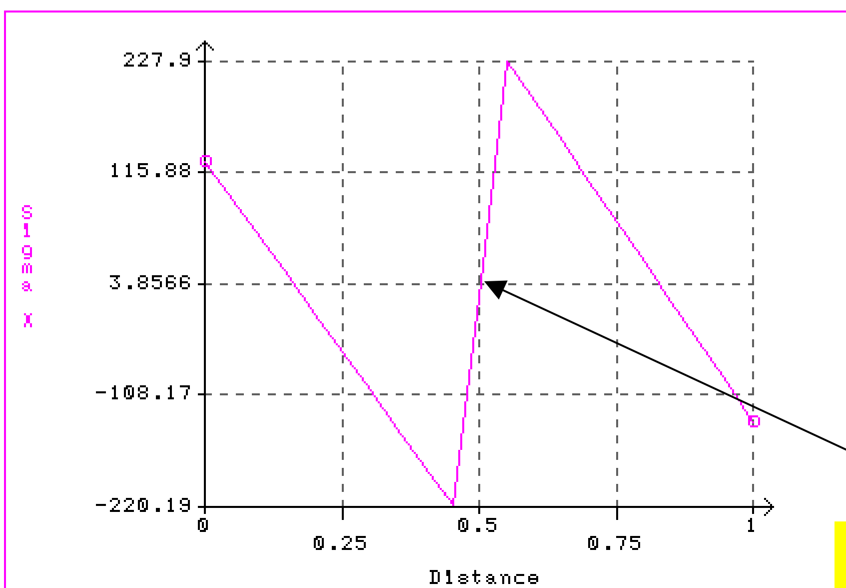
Rezultate: - distribuţia tensiunilor σ_x



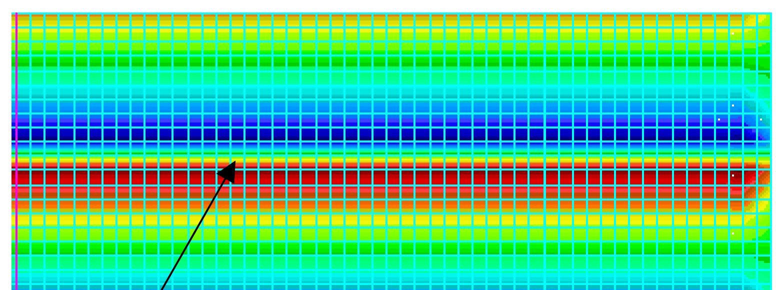


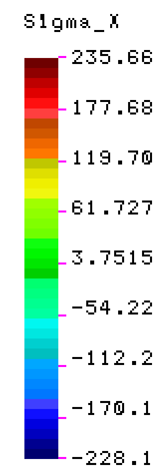
Reprezentarea prin elemente

Reprezentare corectă !



Reprezentarea prin noduri

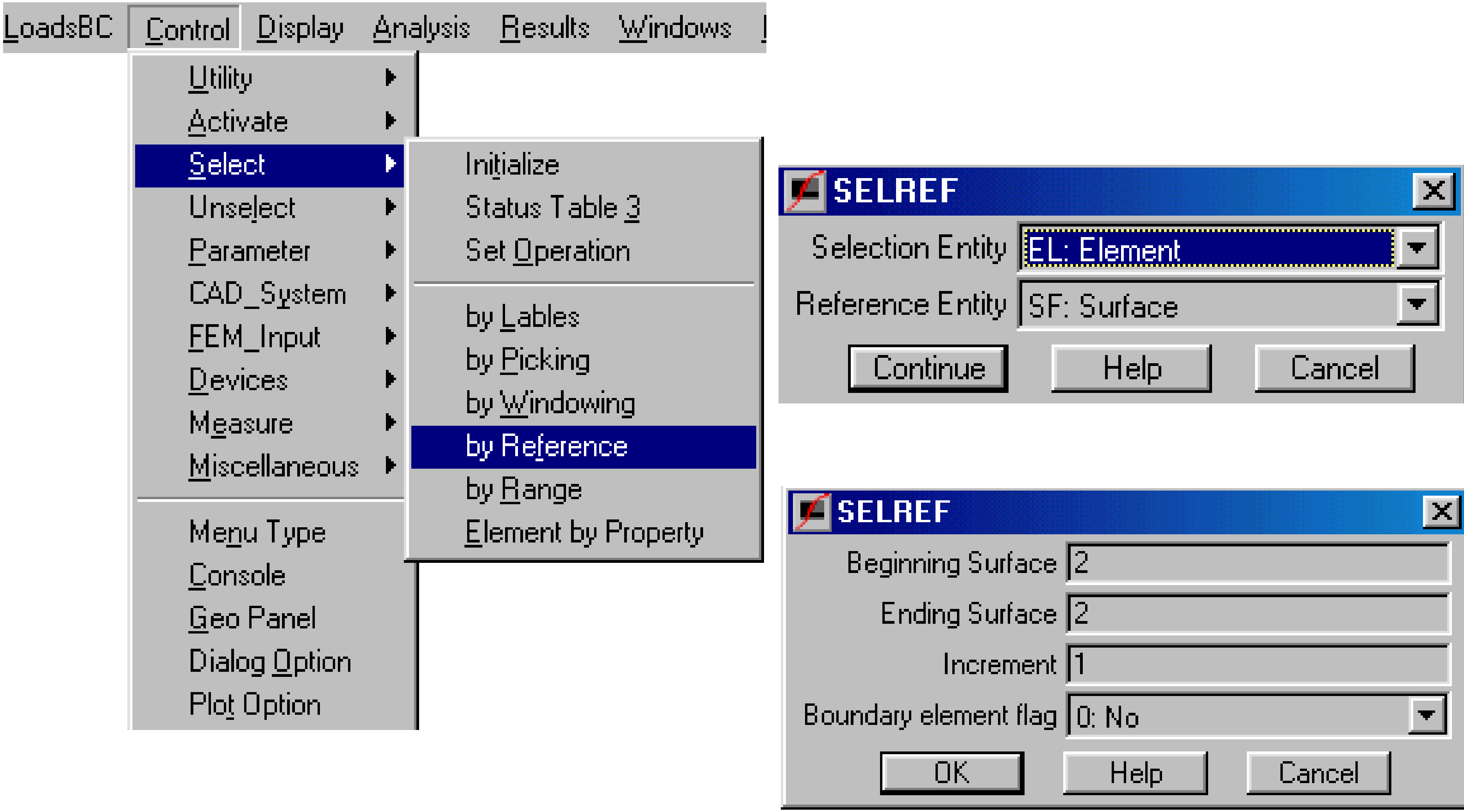




Atenţie - reprezentare eronată la interfaţa !

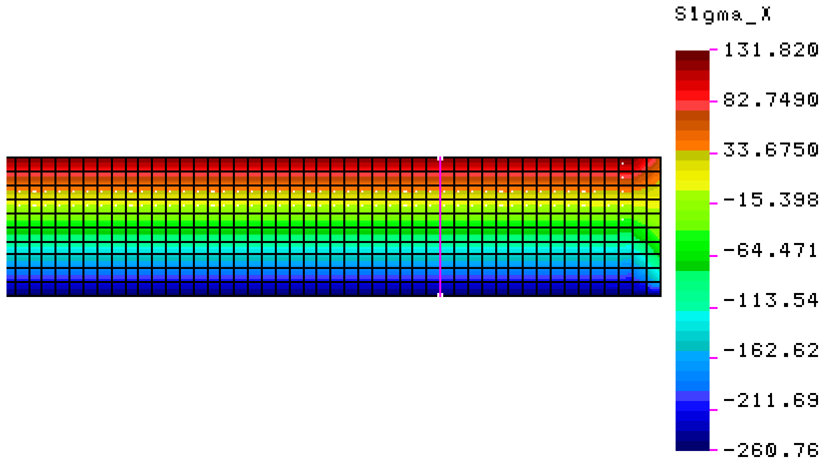
Notă: pt. reprezentarea grafică s-a utilizat opţiunea aspect raţio y/x =10

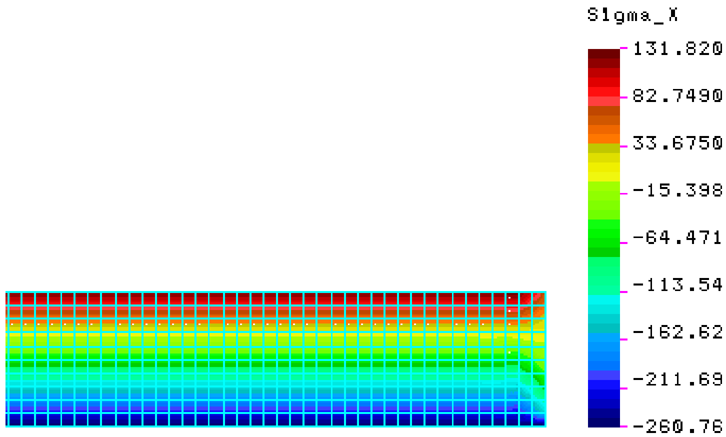
Pentru a obţine reprezentarea corectă a tensiunilor în noduri şi la nivelul interfeţei dintre cele două materiale, se apelează la reprezentarea grafică prin intermediul opţiunii SELECT

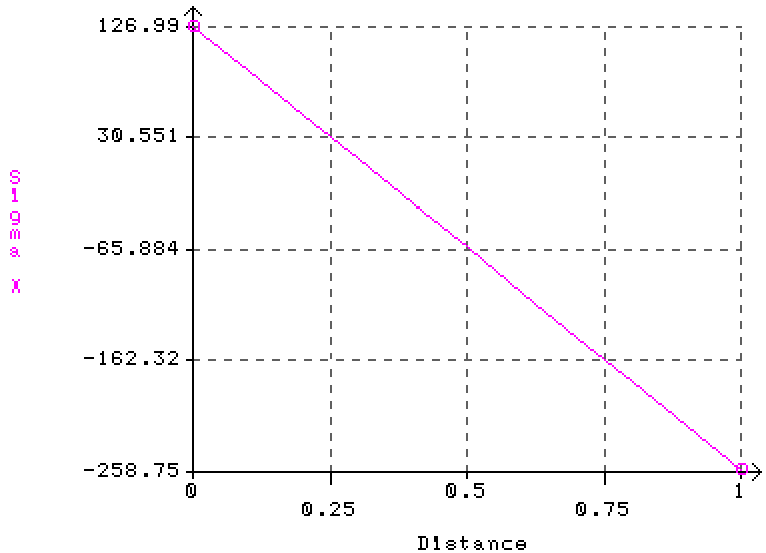


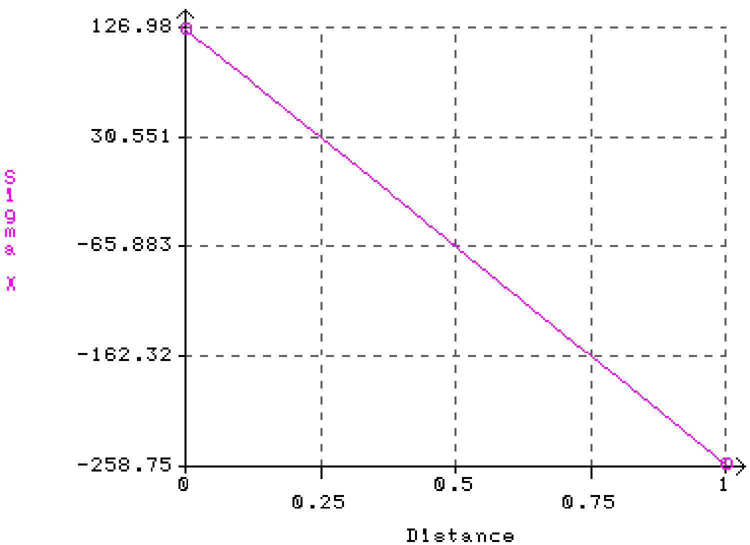
Suprafaţa (materialul 2)

Suprafaţa (materialul 1)









Reprezentare corectă !